

九十四學年四技二專第二次聯合模擬考試

共同考科 數學(B)卷 詳解

數學(B)卷

94-2-3

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	B	A	B	C	C	C	A	D	B	B	A	C	B	D	A	A	B	A	A	C	C	B	C

1. $f(x) = 2(x-3)^2 + 5 = 2x^2 - 12x + 23$

$\therefore a = -12, b = 23$

$2a + b = -24 + 23 = -1$

2. $\theta = \frac{\pi}{3}$

$A = \frac{1}{2}r^2\theta = \frac{1}{2} \cdot 6^2 \cdot \frac{\pi}{3} = 6\pi$

3. $a = 0.456\boxed{4}56\dots$

$b = 0.456\boxed{5}65\dots$

$c = 0.456\boxed{6}66\dots \quad \therefore c > b > a$

4. 設 $f(x) = (x-1)(x-2)Q(x) + (ax+b)$

$f(1) = a + b = -3 \quad f(2) = 2a + b = 2$

$\therefore a = 5, b = -8$

$\therefore r(x) = 5x - 8 \quad \therefore r(-3) = -23$

5. 判別式 $D = (8k+1)^2 - 4 \cdot 2k \cdot 8k > 0$

$64k^2 + 16k + 1 - 64k^2 > 0$

$16k > -1 \quad \therefore k > -\frac{1}{16}$

6. $f(x) = 6x^3 - 13x^2 + 4$

$f(x) = (x-2)(6x^2 - x - 2) = (x-2)(2x+1)(3x-2)$

$\therefore b = 1, c = -2$

$\therefore b+c = -1$

7. 令二根為 $\alpha, 2\alpha$

$\therefore \alpha + 2\alpha = -p \quad \therefore p = -3\alpha$

$\alpha \cdot 2\alpha = q \quad \therefore q = 2\alpha^2 \quad \therefore 2p^2 = 9q$

8. 令所求直線方程式為 $3x + 2y = k$

(2,2) 代入 $\therefore k = 10 \quad \therefore 3x + 2y = 10$

9. $2^{2x} + 2^{x+2} - 5 = 0 \quad (2^x)^2 + 4 \cdot 2^x - 5 = 0$

$\therefore (2^x + 5)(2^x - 1) = 0$

$\therefore 2^x = -5$ (不合) $2^x = 1 \quad \therefore x = 0$

$\therefore a^2 - 3a + 5 = 5$

10. 原式 $= \log_2 2^3 - \log_3 3^{-3} + \log_5 (5^3)^{\frac{1}{3}}$
 $= 3 + 3 + 1 = 7$

11. $\log(\frac{1}{5})^n < \log 10^{-6} \quad n \log \frac{2}{10} < -6$

$n(\log 2 - \log 10) < -6 \quad \therefore -0.6990n < -6$

$\therefore n > \frac{6}{0.6990} \doteq 8.58 \quad \therefore n = 9$

12. $(a^2b, a-b) \Rightarrow (-,+)$

∴ 第二象限

$$\begin{aligned} 13. \sum_{k=2}^{\infty} \frac{2^{k-1} + 3^{k+1}}{5^k} &= \sum_{k=2}^{\infty} \frac{2^{k-1}}{5^k} + \sum_{k=2}^{\infty} \frac{3^{k+1}}{5^k} \\ &= \frac{1}{5} \sum_{k=2}^{\infty} \left(\frac{2}{5}\right)^{k-1} + 3 \sum_{k=2}^{\infty} \left(\frac{3}{5}\right)^k \\ &= \frac{1}{5} \times \frac{\frac{2}{5}}{1 - \frac{2}{5}} + 3 \times \frac{\frac{27}{25}}{1 - \frac{3}{5}} = \frac{17}{6} \end{aligned}$$

14. 原式 $= \sin 210^\circ + \cos 120^\circ + \tan 315^\circ + \sec 330^\circ$
 $= -\sin 30^\circ - \cos 60^\circ - \tan 45^\circ + \sec 30^\circ$

$= -\frac{1}{2} - \frac{1}{2} - 1 + \frac{2}{\sqrt{3}} = -2 + \frac{2\sqrt{3}}{3}$

15. $\sin 200^\circ$ 為負

$1 - \cos^2 \theta > 0 (\because -1 < \cos \theta < 1)$

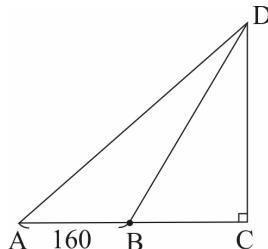
∴ $(\sin 200^\circ, 1 - \cos^2 \theta) \Rightarrow (-,+)$ 在第二象限16. 令山高 $\overline{DC} = h$

$\therefore \overline{AC} = h \quad \therefore \overline{BC} = h - 160$

$\therefore \tan 60^\circ = \frac{h}{h-160} \quad \sqrt{3} = \frac{h}{h-160}$

$\sqrt{3}h - 160\sqrt{3} = h \quad (\sqrt{3}-1)h = 160\sqrt{3}$

$h = \frac{160\sqrt{3}}{\sqrt{3}-1} = \frac{160\sqrt{3}(\sqrt{3}+1)}{2} = 240 + 80\sqrt{3}$

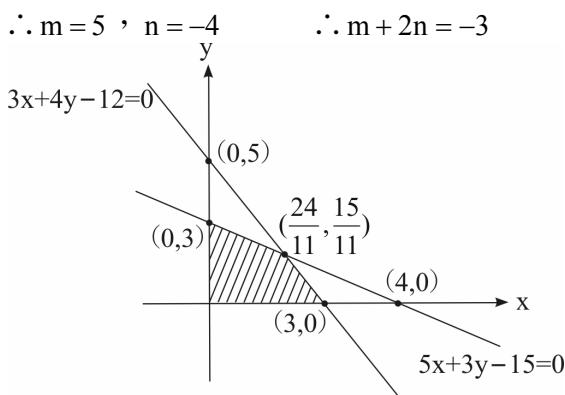


17. $\therefore \Delta ABC = \frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot 8 \cdot 12 \cdot \frac{\sqrt{2}}{2} = 24\sqrt{2}$

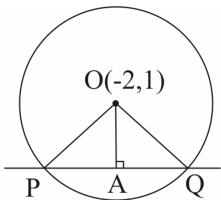
18. $f(0,0) = -1 \quad f(0,3) = -4$

$f(3,0) = 5 \quad f\left(\frac{24}{11}, \frac{15}{11}\right) = 2$



19. 令圓心 $O(a, 0)$ 則 $\sqrt{(a-2)^2 + 4^2} = \sqrt{(a-7)^2 + 1^2}$
 $\therefore a = 3$ 圓心 $(3, 0)$ $r = \sqrt{(3-2)^2 + 4^2} = \sqrt{17}$
 圓方程式 $(x-3)^2 + y^2 = 17 \quad x^2 + y^2 - 6x - 8 = 0$

20. $\overline{OA} = \frac{|-6-4|}{\sqrt{9+16}} = 2$
 $\overline{PA} = \sqrt{3^2 - 2^2} = \sqrt{5} \quad \therefore \overline{PQ} = 2\sqrt{5}$
 ΔOPQ 面積 = $\frac{1}{2} \cdot 2\sqrt{5} \cdot 2 = 2\sqrt{5}$



21. $16 + 9 - 32 + 12 - 5 = 0 \quad \therefore (-4, 3)$ 在圓上
 \therefore 過 $(-4, 3)$ 且與圓相切的直線方程式為

$$-4x + 3y + 8 \cdot \frac{-4+x}{2} + 4 \cdot \frac{3+y}{2} - 5 = 0 \quad y = 3$$

22. $a_1 + b_1 = 100 \quad 99d = (a_{100} + b_{100}) - (a_1 + b_1)$
 $99d = 0 \quad \therefore d = 0$

$$\sum_{i=1}^{100} (a_i + b_i) = 100(a_1 + b_1) = 100 \times 100 = 10000$$

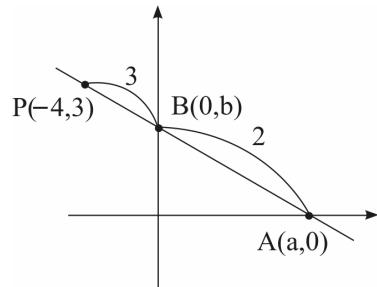
23. $\begin{cases} a + ar^3 = 52 \\ ar + ar^2 = 40 - 52 = -12 \end{cases}$
 $\therefore \begin{cases} a(1+r)(1-r+r^2) = 52 \cdots (1) \\ ar(1+r) = -12 \cdots (2) \end{cases}$

$$\frac{(1)}{(2)} \quad \frac{1-r+r^2}{r} = -\frac{13}{3}$$

$$3r^2 + 10r + 3 = 0 \quad (r+3)(3r+1) = 0$$

$$r = -3, -\frac{1}{3}$$

24. $\because \overline{AP} : \overline{PB} = 5 : 3 \quad \therefore \overline{PB} : \overline{BA} = 3 : 2$
 $\frac{3a-8}{3+2} = 0 \quad a = \frac{8}{3} \quad \frac{0+6}{3+2} = b \quad b = \frac{6}{5}$



方程式 $\frac{x}{8} + \frac{y}{6} = 1$
 $\frac{3}{3} \quad \frac{5}{5}$
 $\therefore 9x + 20y - 24 = 0$

25. $6x - 12 + 4x + 2 > 0$
 $10x > 10, x > 1$